

## **UNIVERSIDAD DE CASTILLA - LA MANCHA**

# **GUÍA DOCENTE**

#### 1. General information

Course: NUMERICAL ANALYSIS OF PDE AND APPROXIMATION Type: ELECTIVE						Code: 310934 ECTS credits: 6		
2351 - MASTER DEGREE PROGRAMME IN PHYSICS AND MATHEMATICS-FISYMAT						Academic year: 2022-23		
Center: 602 - E.T.S. INDUSTRIAL ENGINEERING OF C. REAL						Group(s): 20		
Year: 1						Duration: First semester		
Main language: Sp	econd la	ond language: English						
Use of additional languages:	Friendly: Y							
Web site: Bilingual: N								
Lecturer: DAMIAN CAS	STAÑO TORRIJOS - Group(s): 2	0						
Building/Office	Department	Phone num	ber	Email	Office hours			
Edificio Sabatini / 1.53	MATEMÁTICAS	926051463	51463 Damian.Castano@uclm.es		Consult in Virtual Campus			
Lecturer: MARIA CRUZ	NAVARRO LERIDA - Group(s)	: 20						
Building/Office	Department	Phone number	Email			Office hours		
Margarita Salas/326	MATEMÁTICAS	3469	mai	riacruz.navarro@uclm.es	prior appointment by e-mail			
Lecturer: FRANCISCO PLA MARTOS - Group(s): 20								
Building/Office	Department	Phone number	Email O		Offi	ice hours		
Margarita Salas	MATEMÁTICAS	3468 francisco.pla@uclm.es prior appointment by e-mail						

### 2. Pre-Requisites

Calculus, algebra, differential equations and functional analysis.

#### 3. Justification in the curriculum, relation to other subjects and to the profession

Partial differential equations are the main tool for modeling in science and technology. Only a few of these equations have an analytical solution. For this reason, numerical resolution is essential for scientific progress. To acquire knowledge on numerical analysis is relevant in an Applied Mathematics Master of Science.

4. Degree compete	ences achieved in this course
Course competenc	es
Code	Description
CB06	Possess and understand knowledge that provides a basis or opportunity to be original in the development and / or application of ideas, often in a research context.
CB07	Apply the achieved knowledge and ability to solve problems in new or unfamiliar environments within broader (or multidisciplinary) contexts related to the area of study
CB08	Be able to integrate knowledge and face the complexity of making judgments based on information that, being incomplete or limited, includes reflections on social and ethical responsibilities linked to the application of knowledge and judgments
CB09	Know how to communicate the conclusions and their supported knowledge and ultimate reasons to specialized and non-specialized aud non
CB10	Have the learning skills which allow to continue studying in a self-directed or autonomous way
CE01	Solve physical and mathematical problems, planning their solutions based on the available tools and time and resource constraints
CE02	Develop the ability to decide the appropriate techniques to solve a specific problem with special emphasis on those problems associated with the Modeling in Science and Engineering, Astrophysics, Physics, and Mathematics
CE05	Know how to obtain and interpret physical and/or mathematical data that can be applied in other branches of knowledge
CE07	Ability to understand and apply advanced knowledge of mathematics and numerical or computational methods to problems of biology, physics and astrophysics, as well as to build and develop mathematical models in science, biology and engineering
CE08	Ability to model, interpret and predict from experimental observations and numerical data
CG03	Present publicly the research results or technical reports, to communicate the conclusions to a specialized court, interested persons or organizations, and discuss with their members any aspect related to them
CG04	Know how to communicate with the academic and scientific community as a whole, with the company and with society in general about Physics and/or Mathematics and its academic, productive or social implications
CT03	Develop critical reasoning and the ability to criticize and self-criticize
CT05	Autonomous learning and responsibility (analysis, synthesis, initiative and teamwork)

5. Objectives or Learning Outcomes

## Course learning outcomes

Description

Understand the specific characteristics of the elliptic, parabolic and hyperbolic equations which be solved by numerical methods Know and understand the basic concepts of consistency, stability and convergence of a numerical scheme in this context, as well as their interrelation Gain the ability to solve a specific problem as a team: from the choice of an appropriate method to the oral and written presentation of the results obtained after its implementation

Learn to use some tools of Basic Analysis and Functional Analysis to carry out the numerical analysis of a method

Know some software tools that allow to completely solve a problem in the computer, which entails know how to program, generate a computational mesh, apply the appropriate calculus module and visualize the numerical solution. Practical problem solving.

Interpretation of the obtained numerical solution and critical judgment of its quality. Relation with the applied science referred to

Understand the theoretic design of finite element, finite difference, finite and spectral volume methods, from known analytic techniques (variational formulations, Taylor developments, integration formulas by parts).

#### 6. Units / Contents

#### Unit 1: Finite Differences

- Unit 1.1 Boundary value problems
- Unit 1.2 Parabolic evolution problems
- Unit 1.3 Error analysis
- Unit 1.4 Practice exercises

### **Unit 2: Spectral Methods**

- Unit 2.1 Fourier approximation
- Unit 2.2 Orthogonal polynomial approximation
- Unit 2.3 Practice exercises

#### **Unit 3: Finite Elements**

- Unit 3.1 Formulation and error analysis
- Unit 3.2 Effective implementation
- Unit 3.3 Practice exercises

#### **Unit 4: Finte Volumes and Finite Differences**

Unit 4.1 Finite Differences for hyperbolic evolution problems

- Unit 4.2 Formulation and error analysis
- Unit 4.3 Effective implementation
- Unit 4.4 Practice exercises

#### Unit 5: Courses and seminars

7. Activities, Units/Modules and Methodology							
Training Activity	Methodology	Related Competences (only degrees before RD 822/2021)	ECTS	Hours	As	Com	Description
Class Attendance (theory) [ON- SITE]	Lectures		1.5	37.5	Y		Theoretical development of the course contents and resolution of exercises
Computer room practice [ON-SITE]	Problem solving and exercises		0.7	17.5	Y	Y	Troubleshooting in the computer room
Individual tutoring sessions [ON- SITE]			0.2	5	Y	N	Individual tutorials for resolution of doubts
Writing of reports or projects [OFF- SITE]	Self-study		3.6 90 Y Resolution of problems and Y preparation of reports and work the contents of the course		preparation of reports and works on		
Total:			6	150			
Total credits of in-class work: 2.4				Total class time hours: 60			
Total credits of out of class work: 3.6				Total hours of out of class work: 90			

As: Assessable training activity

Com: Training activity of compulsory overcoming (It will be essential to overcome both continuous and non-continuous assessment).

8. Evaluation criteria and Grading System							
Evaluation System	Continuous assessment	Non- continuous evaluation*	Description				
Assessment of active participation	20.00%	10.00%	Active participation in solving problems				
Assessment of activities done in the computer labs	30.00%	20.00%	Resolution of practice exercises in the computer lab				
Practicum and practical activities reports assessment	50.00%	70.00%	Delivery of proposed works				
Total	: 100.00%	100.00%					

According to art. 4 of the UCLM Student Evaluation Regulations, it must be provided to students who cannot regularly attend face-to-face training activities the passing of the subject, having the right (art. 12.2) to be globally graded, in 2 annual calls per subject, an ordinary and an extraordinary one (evaluating 100% of the competences).

#### Evaluation criteria for the final exam:

Continuous assessment:

Assessment of active participation, work in the computer lab, and autonomous work

#### Non-continuous evaluation:

Evaluation criteria not defined

Hours	hours
Unit 1 (de 5): Finite Differences	
Activities	Hours
Class Attendance (theory) [PRESENCIAL][Lectures]	8
Computer room practice [PRESENCIAL][Problem solving and exercises]	4
Individual tutoring sessions [PRESENCIAL][]	1
Writing of reports or projects [AUTÓNOMA][Self-study]	20
Unit 2 (de 5): Spectral Methods	
Activities	Hours
Class Attendance (theory) [PRESENCIAL][Lectures]	8
Computer room practice [PRESENCIAL][Problem solving and exercises]	4
Individual tutoring sessions [PRESENCIAL][]	1
Writing of reports or projects [AUTÓNOMA][Self-study]	20
Unit 3 (de 5): Finite Elements	
Activities	Hours
Class Attendance (theory) [PRESENCIAL][Lectures]	8
Computer room practice [PRESENCIAL][Problem solving and exercises]	4
Individual tutoring sessions [PRESENCIAL][]	1
Writing of reports or projects [AUTÓNOMA][Self-study]	20
Unit 4 (de 5): Finte Volumes and Finite Differences	
Activities	Hours
Class Attendance (theory) [PRESENCIAL][Lectures]	8
Computer room practice [PRESENCIAL][Problem solving and exercises]	4
Individual tutoring sessions [PRESENCIAL][]	1
Writing of reports or projects [AUTÓNOMA][Self-study]	20
Unit 5 (de 5): Courses and seminars	
Activities	Hours
Class Attendance (theory) [PRESENCIAL][Lectures]	5.5
Computer room practice [PRESENCIAL][Problem solving and exercises]	1.5
Individual tutoring sessions [PRESENCIAL][]	1
Writing of reports or projects [AUTÓNOMA][Self-study]	10
Global activity	
Activities	hours
Computer room practice [PRESENCIAL][Problem solving and exercises]	17.5
Individual tutoring sessions [PRESENCIAL][]	5
Writing of reports or projects [AUTÓNOMA][Self-study]	90
Class Attendance (theory) [PRESENCIAL][Lectures]	37.5
	Total horas: 150

10. Bibliography and Sources						
Author(s)	Title/Link	Publishing house	Citv	ISBN	Year	Description
A.Quarterioni, R.Sacco, F.Saleri	Numerical Mathematics	Springer-Verlag			2000	
C. Bernardi and Y. Maday	Approximations spectrales de problemes aux limites elliptiques	Springer			1992	
C. Canuto, M.Y. Hussaini, A. Quarteroni and T.A. Zang	Spectral Methods for Fluid Dynamics	Springer			1988	
C. Johnson	Numerical solution of P.D.E. by the Finite Element Method	Cambridge University Press			1987	
D.F.Griffiths, A.R.Mitchell	The finite difference method in partial differential equation	John Wiley			1980	
E. Godlewski, P.A. Raviart	Hyperbolic systems of conservation laws	Ellipses			1991	
E.Godlewski, P.A. Raviart	Numerical Approximation of Hyperbolic Systems of Conservation Laws	Springer-Verlag			1996	
G.D. Smith	Numerical Solution of Partial Differential Equations: Finite Difference Methods	Oxford University Press			1985	
G.F.Forsythe, W.R.Wasow	Finite difference methods for partial differential equations	John Wiley			1960	
J. C. Strikwerda	Finite difference Schemes and Partial Differential	Pacific Grove, CA: Wadsworth and Brooks			1989	
J.H.Mathews, K.D. Fink	Métodos Numéricos con MATLAB	Prentice-Hall			2000	
J.M. Sanz-Serna	Fourier techniques in numerical methods for evolutionary problems. 3RD Granada Seminar on Computational Physics	Springer			1995	
L.N. Trefethen	Spectral methods in Matlab	SIAM			2000	
O.C. Zienkiewicz	The Finite Element Method in Engineering Science	McGraw-Hill			1971	
P.G.Ciarlet	The finite element method for	North Holland			1978	

R. LeVeque	ellintic problems Finite Volume Methods for Hyperbolic Problems	Cambridge Univesity Press		2002	
R.B. Richtmyer, K.W. Morton	Difference methods for initial- value problems	John Wiley & Sons		1967	
R.G. Voigt, D. Gotlieb and M.Y. Hussaini	Spectral Methods for Partial Differential Equations	SIAM		1984	
S. Nakamura	Análisis Numérico y visualización gráfica con MATLAB	Pearson Educación/Prentice- Hall Hispanoamerica		1997	
Gwynne A. Evans, G. Evans, G. A Evans, Jonathan M. Blackledge, J. Blackledge, Peter D. Yardley, P. Yardley	Numerical Methods for Partial Differential Equations	Springer		2000	
Stig Larsson, Vidar Thomee	Partial Differential Equations with Numerical Methods	Springer		2014	
Randall LeVeque	Finite difference methods for ordinary and partial differential equations	SIAM		2007	
E. Romera, M.C. Boscá, F. Arias, F.J. Gálvez, J.I. Porras	Métodos Matemáticos: Problemas de Espacios de Hilbert, Operadores Lineales y Espectros	Paraninfo		2013	
C.H. Edwards, D. E. Penney	Ecuaciones diferenciales y problemas con valores en la frontera. Cómputo y modelado (4ª Ed)	Pearson		2009	
Frederic Hecht	FreeFEM Documentation. Release 4.6			2021	Pagina oficial de FreeFEM
Roberto Font, Francisco Periago	https://freefem.org/ The Finite Element Method with FreeFem++ for beginners		1933-2823	2013	
Sandip Mazumder	Numerical Methods for Partial Differential Equations: Finite Difference and Finite Volume Methods	Elsevier Science		2016	
P.M. Gresho, R.L. Sani	Incompressible Flow and the Finite Element. Volume One. Advection-Diffusion	John Wiley & Sons	0471492493	2000	
J.N. Reddy	An Introduction to the Finite Element Method	MCGRAW HILL SERIES IN MECHANICAL ENGINEERING		2005	
P.M. Gresho, R.L. Sani	Incompressible Flow and the Finite Element. Volume Two. Isothermal Laminar Flow	John Wiley & Sons	0471492507	2000	
Haïm Brézis	Análisis Funcional	Alianza Universidad Textos	84-206-8088-5	1983	