UNIVERSIDAD DE CASTILLA - LA MANCHA



GUÍA DOCENTE

1. General information

Web site:					Bilingual: N		
Use of additional languages:				English	Friendly: N		
Main language: English Second language: Spanish					anguage: Spanish		
Year:			Duration: First semester				
Center: 603 - E.T.S. CIVIL ENGINEERS OF CR					Group(s): 20		
2343 - MASTERS DEGREE PROGRAMME IN ENGINEERING OF ROADS, CANALS AND PORTS			Academic year: 2021-22				
Туре:	Type: CORE COURSE				ECTS credits: 9		
Course: MATHEMATICAL MODELING IN CIVIL ENGINEERING					Code: 310800		

2. Pre-Requisites

The following prerequisites are essential or highly recommended in order for the student to follow, without significant conceptual gaps, the contents of the course:

- Knowledge of single variable and multiple variable calculus (both differential and integral). This is essential.
- Knowledge of how to solve linear systems and acquaintance with elementary linear algebra properties. This is essential.
- Knowledge of basic analytical methods to solve elementary differential equations (both ordinary and partial). This is
 essential.
- Knowledge of basic interpolation and approximation techniques for functions and data. Highly recommended.
- Familiarity with MATLAB software. Highly recommended. Other programming languages oriented to numerical computing are also recommended (e.g. Python, Octave, Julia, Mathematica, etc).
- Acquaintance with fundamental equations and models arising in Mechanics of Materials, Continuous Media and Hydrology. Highly recommended.

3. Justification in the curriculum, relation to other subjects and to the profession

Nowadays, nearly all engineering companies and firms worldwide utilise modelling software to deal with projects, from small ones to big ones. Civil engineering students at the master level should be able not only to acquire the ability to use those complex (and very often expensive) programs but also to understand the underlying conceptual elements that make up those programs. Moreover, developing the skills to construct mathematical models (from simple to very technical ones) that can solve problems posed in a non-mathematical fashion, specially within the professional engineering scenario, can make a big difference between just a competent engineer and a truly *super-cruncher* engineer. It is frequently heard that in the professional context most civil engineers only employ a very basic knowledge of mathematics. While in most routine situations it is not necessary to have a great deal of mathematical knowledge to solve civil engineering problems (one may resort to well-known rules of thumb or to the use of the previously mentioned specific software, etc), having a sound background in mathematical modelling capabilities can make a huge impact when the time comes to really find both creative an innovative solutions to new and challenging problems.

The aim of this course is to provide the necessary tools to master-level students in order to acquire and develop mathematical modelling abilities useful for the professional civil engineering. We will start from elementary numerical methods (some of which were already studied during the Degree of Civil Engineering) and then move on to more advanced techniques to solve problems which, quite often, will be proposed in a non-mathematical context and with minimal information. Our modelling software of choice will be MATLAB, although other numerically-oriented software such as Python, Octave or Mathematica can be used as well. It is also worth mentioning that part of the contents of this course will be very useful in other master courses such as Continuum Mechanics and Materials Science, Coastal Engineering, Geotechnical Engineering, Hydraulic Works and Hydroelectric Exploitation, Transport Economy and, specially relevant for the Final Master Thesis. The far reaching goal is that every student, when given suitable practical scenarios, should be able to become proficient in constructing his/her own mathematical models and to solve them by means of the studied methods and techniques or even new ones developed by him/herself if required.

4. Degree com	npetences achieved in this course
Course compe	tences
Code	Description
AFC1	Ability to address and solve advanced mathematical engineering problems, from problem solving to formulation development and implementation in a computer program. In particular, the ability to formulate, program and apply advanced analytical and numerical models for calculation, design, planning and management, as well as the ability to interpret the results obtained, in the context of civil engineering.
CB06	Possess and understand knowledge that provides a basis or opportunity to be original in the development and / or application of ideas, often in a research context.
CB07	Apply the achieved knowledge and ability to solve problems in new or unfamiliar environments within broader (or multidisciplinary)

CB09	KNOW No Work Communicate the Controlusions and their supported knowledge and ultimate reasons to specialized and non-specialized audiences in a clear and unambiguous way
CB10	Have the learning skills which allow to continue studying in a self-directed or autonomous way
G01	Scientific-technical and methodological capacity for the continuous recycling of knowledge and the exercise of the professional functions of consultancy, analysis, design, calculation, project, planning, leadership, management, construction, maintenance, conservation and exploitation in the fields of civil engineering.
G17	Adequate knowledge of the scientific and technological aspects of mathematical, analytical and numerical methods of engineering, fluid mechanics, mechanics of continuous means, structural calculations, ground engineering, maritime engineering, water resources and linear works.
G18	Ability to participate in research projects and scientific and technological collaborations within its thematic area, in interdisciplinary contexts and, where appropriate, with a high knowledge transfer component.
G19	Knowledge of the latest developments and applications of technology to civil engineering in all its fields, as well as its new challenges.
G21	Ability to apply optimization tools to aid decision making, as well as to discern exploitation proposals compatible with the constraints and peculiarities of the built infrastructure.
G25	Ability to identify, measure, enunciate, analyse, diagnose and scientifically and technically describe a civil engineering problem
G27	Ability to communicate in a second language.
G28	Ability to work in an international context.
G29	Management capacity and teamwork.

5. Objectives or Learning Outcomes

Course learning outcomes

Description

Students mathematically formulate and quantitatively solve a problem involving (ordinary and/or partial) differential equations using analytical techniques and/or numerical methods.

Students develop and program codes to implement the numerical methods studied to solve ordinary and/or partial differential equations that occur in the field of civil engineering.

Students address computationally intensive problems efficiently.

Students can use estimation techniques for quantities and associated errors.

contexts related to the area of study.

Reinforcement of the students' deductive reasoning capacity.

Students solve basic problems of optimization and optimal control that arise in the planning and management of civil engineering.

Increase in the students' capacity for abstraction.

Students use software platforms to numerically address problems arising in the field of civil engineering.

6. Units / Contents

Unit 1: Introduction to Mathematical Modelling in Civil Engineering

Unit 2: Introduction to Platforms for Advanced Numerical Computation: MATLAB

Unit 3: Review of Basic Numerical Methods

Unit 4: Numerical Solution of Ordinary Differential Equations

Unit 5: Numerical Solution of Partial Differential Equations

Unit 6: Optimization Methods in Civil Engineering

7. Activities, Units/Modules and Methodology

Training Activity	Methodology	Related Competences (only degrees before RD 822/2021)	ECTS	Hours	As	Com	Description
Class Attendance (theory) [ON- SITE]	Lectures	AFC1 CB06 CB07 CB09 CB10 G01 G17 G18 G19 G21 G25 G27 G28 G29	1.36	34	N	-	be presented in the classroom through transparencies/blackboard. Notes and bibliographic excerpts wil be made available in the Campus Virtual.
Problem solving and/or case studies [ON-SITE]	Project/Problem Based Learning (PBL)	AFC1 CB06 CB07 CB09 CB10 G01 G17 G18 G19 G21 G25 G27 G28 G29	0.6	15	Y	N	Following every lecture, problem set will be proposed to the students to b solved and presented during class. These sessions are at the heart of th course as they provide the required skills to assimilate the contents of the course and facilitate the preparation of the exam. In order for this training activity to be assessable, it will be necessary for the student to present individually, during the sessions (which will be reported in advance throughout the course), to the rest of the class the partial/complete solutions to the problems addressed by him/her. Depending on the level of difficulty of the problems (which will be specified in advance in the problem collections), as well as on the performance shown by the student, a score will be assigned for each problem presented.

	Total credi	ts of out of class work: 6.28				Total hours of out of class work: 15
	Total ci	redits of in-class work: 2.72				Total class time hours: 6
	•	Total:	9	225		
/riting of reports or projects [OFF ITE]	Project/Problem Based Learning (PBL)	AFC1 CB06 CB07 CB09 CB10 G01 G17 G18 G19 G21 G25 G27 G28 G29	1.6	40	N	J -
n-line debates and forums [OFF- ITE]	Online Forums	CB06 CB07 CB09 G01 G17 G18 G19 G27 G28 G29	0.2	5	N	y -
tudy and Exam Preparation [OFF ITE]	Self-study	AFC1 CB06 CB07 CB09 CB10 G01 G17 G18 G19 G21 G25 G27 G28 G29	3.68	92	N	N -
nal test [ON-SITE]	Assessment tests	CB06 CB07 CB09 G01 G17 G18 G19 G21 G25 G27 G28 G29	0	0	Y	Students will have two opportunities to take the exam: the Ordinary and the Extraordinary calls. In either of the two calls, the test will have the same structure: it will consist of a questionnaire in which the student can choose a subset of questions from the total proposed followed by three-four development problems to be completed within about 4 hours. Any of these exams will incorporate the contents from the entire course. Since the exams will require differen competences aimed at solving problems, it is highly advisable that students attend the problem session regularly or, independently, try to solve as many as they can. This activity takes place outside the class period.
racticum and practical activities port writing or preparation [OFF- ITE]	Self-study	AFC1 CB06 CB07 CB09 CB10 G01 G17 G18 G19 G21 G25 G27 G28 G29	0.8	20	N	they recoverable.
omputer room practice [ON-SITE] (PBL)	AFC1 CB06 CB07 CB09 CB10 G01 G17 G18 G19 G21 G25 G27 G28 G29	0.76	19	Y	 solve computational problems using the studied numerical methods. Students are encouraged to bring their own laptops to the computer sessions, which will take place after completing each lesson (the specific dates will be announced in advance during the course). Students will learn how to use at least one programming environment: preferentially MATLAB. Open source environments, such as Python, Julia Maxima or Octave will also be accepted if the students are proficier in their use, although less support will be provided. During these computer sessions, a computational problem, chosen within the realm of Civil Engineering, will be proposed. This problem will be solved either individually or in small teams (the modality and time available will be previously specified). Most of the computational problems will nee to be completed during class. Students must send their developed programs through Virtual Campus. These

As: Assessable training activity Com: Training activity of compulsory overcoming (It will be essential to overcome both continuous and non-continuous assessment).

8. Evaluation criteria and Grading System							
Evaluation System	Continuous assessment	Non- continuous evaluation*	Description				
			Students will have to actively solve and present individually, during the problem sessions, their solutions to some of the proposed problems chosen within the proposed collections. The number of problems to be solved by each student throughout the course will depend on their level of difficulty (it				

Assessment of problem solving and/or case studies	15.00%	0.00%	will be indicated in advance when providing the collections). The methodology used and the degree of performance in the resolution and presentation shown by the student will be assessed. The weighting of the grade achieved in solving problems in the overall grade for the subject is 20%. This assessable activity is recovered in the final exam (either the Ordinary / Extraordinary calls).
Final test	50.00%	100.00%	Ordinary/Extraordinary exams. The exam, in any of the Ordinary/Extraordinary calls, will have the same structure: it will consist of a short questionary followed by three-four full- development problems to be completed within 4 hours. Any of these exams will be global and, therefore, will include all the contents of the course. It is important to emphasise that a minimum grade will be required for the final exam (either the Ordinary/Extraordinary call) so as to take into account the assessment from the other activities as well. This minimum grade is 5/10. If this minimum grade is not reached in any of the two exams (Ordinary/Extraordinary), the student will not pass the course.
Assessment of activities done in the computer labs	35.00%	0.00%	The programming codes submitted by the students (via web upload through the Campus Virtual) for each assigned problem will be evaluated. Both interaction with the professor and active participation during these sessions will also be valued. This compulsory training activity is not recoverable. The weighting of the mark achieved in the computer sessions with computers in the overall grade for the subject is 30%.
Total:	100.00%	100.00%	

According to art. 4 of the UCLM Student Evaluation Regulations, it must be provided to students who cannot regularly attend face-to-face training activities the passing of the subject, having the right (art. 12.2) to be globally graded, in 2 annual calls per subject, an ordinary and an extraordinary one (evaluating 100% of the competences).

Evaluation criteria for the final exam:

Continuous assessment:

Students will have two opportunities to pass the course: in the Ordinary and Extraordinary calls. The exam, in either of the two calls, will present the same structure: it will consist of a questionnaire of short problems to choose from followed by three/four development problems. Any of these exams will include all the thematic contents seen in the course. In order to take into consideration the score obtained by the student in the final exam with the rest of the assessable activities (individual problem solving and computational practices), the student must obtain at least 4 points out of 10 in either of the two exams. The final grade for the course will consist of 50% of the exam, 30% of the computational practices and 20% of the individual resolution of problems or cases with presentation during the class. The course will be passed if the average grade is at least 5 points out of 10.

The grades corresponding to the following activities: problem solving and computer practices, all carried out in the previous academic year, are kept for the next one provided that at least 4 points out of 10 had been obtained in each of them.

Non-continuous evaluation:

Unless stated otherwise, students are in the continuous assessment system. Those who decide to opt for non-continuous assessment must notify the professor of the subject before the end of the class period and may only do so if their participation in the assessable activities (from the continuous assessment system) does not reach or exceeds the 50% of the total assessable activities of the course.

Students will have to do a global exam that will include all the course contents and competences. The structure of this exam will be significantly broader than that of the Ordinary/Extraordinary exams. To pass the course, the student must obtain at least 5 points out of 10 in this exam, which will represent 100% of his/her final grade.

Specifications for the resit/retake exam:

In the Extraordinary call, each student would be in the same evaluation system (continuous or non-continuous) as in the Ordinary call. The same criteria will be applied as in the Ordinary call.

Specifications for the second resit / retake exam:

The student will have to do a global exam that will include all the course contents and competences. The structure of this exam will be significantly broader than that of the Ordinary/Extraordinary exams. To pass the course, the student must obtain at least a 5 points out of 10 in this exam, which will represent 100% of his/her final grade.

9. Assignments, course calendar and important dates	
Not related to the syllabus/contents	
Hours hours	
Unit 1 (de 6): Introduction to Mathematical Modelling in Civil Engineering	
Activities	Hours
Class Attendance (theory) [PRESENCIAL][Lectures]	2
Problem solving and/or case studies [PRESENCIAL][Project/Problem Based Learning (PBL)]	2
Study and Exam Preparation [AUTÓNOMA][Self-study]	10
On-line debates and forums [AUTÓNOMA][Online Forums]	1
Writing of reports or projects [AUTÓNOMA][Project/Problem Based Learning (PBL)]	8
Unit 2 (de 6): Introduction to Platforms for Advanced Numerical Computation: MATLAB	
Activities	Hours
Class Attendance (theory) [PRESENCIAL][Lectures]	2
Problem solving and/or case studies [PRESENCIAL][Project/Problem Based Learning (PBL)]	1
Computer room practice [PRESENCIAL][Project/Problem Based Learning (PBL)]	2
Study and Exam Preparation [AUTÓNOMA][Self-study]	12
Writing of reports or projects [AUTÓNOMA][Project/Problem Based Learning (PBL)]	2
Unit 3 (de 6): Review of Basic Numerical Methods	

Activities	Hours
Class Attendance (theory) [PRESENCIAL][Lectures]	9
Problem solving and/or case studies [PRESENCIAL][Project/Problem Based Learning (PBL)]	4
Computer room practice [PRESENCIAL][Project/Problem Based Learning (PBL)]	4
Practicum and practical activities report writing or preparation [AUTÓNOMA][Self-study]	5
Study and Exam Preparation [AUTÓNOMA][Self-study]	20
On-line debates and forums [AUTÓNOMA][Online Forums]	1
Writing of reports or projects [AUTÓNOMA][Project/Problem Based Learning (PBL)]	8
Unit 4 (de 6): Numerical Solution of Ordinary Differential Equations	
Activities	Hours
Class Attendance (theory) [PRESENCIAL][Lectures]	6
Problem solving and/or case studies [PRESENCIAL][Project/Problem Based Learning (PBL)]	3
Computer room practice [PRESENCIAL][Project/Problem Based Learning (PBL)]	4
Practicum and practical activities report writing or preparation [AUTÓNOMA][Self-study]	5
Study and Exam Preparation [AUTÓNOMA][Self-study]	14
On-line debates and forums [AUTÓNOMA][Online Forums]	1
Writing of reports or projects [AUTÓNOMA][Project/Problem Based Learning (PBL)]	6
Unit 5 (de 6): Numerical Solution of Partial Differential Equations	
Activities	Hours
Class Attendance (theory) [PRESENCIAL][Lectures]	8
Problem solving and/or case studies [PRESENCIAL][Project/Problem Based Learning (PBL)]	3
Computer room practice [PRESENCIAL][Project/Problem Based Learning (PBL)]	6
Practicum and practical activities report writing or preparation [AUTÓNOMA][Self-study]	5
Study and Exam Preparation [AUTÓNOMA][Self-study]	20
On-line debates and forums [AUTÓNOMA][Online Forums]	1
Writing of reports or projects [AUTÓNOMA][Project/Problem Based Learning (PBL)]	8
Unit 6 (de 6): Optimization Methods in Civil Engineering	
Activities	Hours
Class Attendance (theory) [PRESENCIAL][Lectures]	7
Problem solving and/or case studies [PRESENCIAL][Project/Problem Based Learning (PBL)]	2
Computer room practice [PRESENCIAL][Project/Problem Based Learning (PBL)]	3
Practicum and practical activities report writing or preparation [AUTÓNOMA][Self-study]	5
Study and Exam Preparation [AUTÓNOMA][Self-study]	16
On-line debates and forums [AUTÓNOMA][Online Forums]	1
Writing of reports or projects [AUTÓNOMA][Project/Problem Based Learning (PBL)]	8
Global activity	
Activities	hours
Class Attendance (theory) [PRESENCIAL][Lectures]	34
Problem solving and/or case studies [PRESENCIAL][Project/Problem Based Learning (PBL)]	15
Writing of reports or projects [AUTÓNOMA][Project/Problem Based Learning (PBL)]	40
Computer room practice [PRESENCIAL][Project/Problem Based Learning (PBL)]	19
Practicum and practical activities report writing or preparation [AUTÓNOMA][Self-study]	20
Study and Exam Preparation [AUTÓNOMA][Self-study]	92
On-line debates and forums [AUTÓNOMA][Online Forums]	5
	Total horas: 225

10. Bibliography and Sources						
Author(s)	Title/Link	Publishing house	Citv	ISBN	Year	Description
Sauer, T.	Numerical Analysis	Third edition, Pearson Education		978-0-13-469645-4	2018	Lessons 3, 4 and 5
Siauw, T., and Bayen, A.M.	An Introduction to MATLAB Programming and Numerical Methods for Engineers	Academic Press, Elsevier			2015	Lesson 2
Sioshansi, R., and Conejo, A.J	Optimization in Engineering: Models and Algorithms	Springer			2017	Lesson 6
Hahn, B.D. and Valentine, D.T.	Essential MATLAB for Engineers and Scientists	Seventh edition, Academic Press, Elsevier		978-0-08-102997-8	2019	Lesson 2
Simon, V., Weigand, B., and Gomaa, H.	Dimensional Analysis for Engineers	Springer		978-3-319-52028-5	2017	Lesson 1
Xue, D	Differential Equation Solutions with MATLAB	De Gruyter		978-3-11-067524-5	2020	Lessons 4 and 5
Bashier, E.B.M.	Practical Numerical and Scientific Computing with MATLAB and Python	CRC Press, Taylor & Francis Group		978-0-42-902198-5	2020	Lessons 3 and 4
Woodford, C., and Phillips, C.	Numerical Methods with Worked Examples: MATLAB	Second edition, Springer			2012	Lessons 3 and 6
Wouver, A.V., Saucez, P., and Vilas, C.	Simulation of ODE/PDE Models with MATLAB, OCTAVE and SCÍLAB: Scientific and	Springer			2014	Lessons 4 and 5

Yang, XS.	Epginization Applications and	John Wiley &		2018	Lesson 6
Zienkiewicz, O.C., Taylor, R.L., and		Sons Seventh edition,		2013	Lesson 5
Zhu, J.Z. Mo, J.P.T, Cheung, S.C.P., and Das, R.	Basis and Fundamentals Demystifying Numerical Models: Step-by-Step Modeling of Engineering Systems	Elsevier Elsevier	978-0-08-100975-8	2019	Lesson 1
Arora, J.S.	Introduction to Optimum Design	Fourth edition, Academic Press, Elsevier		2016	Lesson 6
Shankar, P.M.	Differential Equations: A Problem Solving Approach Based on MATLAB	CRC Press, Taylor & Francis Group		2018	Lesson 4
Attaway, S.	MATLAB: A Practical Introduction to Programming and Problem Solving	Fifth edition, Elsevier	978-0-12-815479-3	2019	Lesson 2
Barnes, B., and Fulford, G.R.	Mathematical Modelling with Case Studies Using Maple and MATLAB			2015	Lesson 1
Belegundu, A.D., and Chadrupatla T.R.	a, Optimization Concepts and Applications in Engineering	Third edition, Cambridge University Press	978-1-108-42488-2	2019	Lesson 6
Bungartz, HJ., Zimmer, S., Buchholz, M., and Pflüger, D.	Modeling and Simulation: An Application-Oriented Introduction	Springer-Verlag		2014	Lesson 1
Burden, R.L., Faires, J.D., and Burden, A.M.	Numerical Analysis	Tenth edition, Brooks/Cole Cengage Learning		2016	Lessons 3 and 4
Butcher, J.C.	Numerical Methods for Ordinary Differential Equations	Third edition, John Wiley & Sons		2016	Lesson 4
Bober, W	MATLAB Essentials A First Course for Engineers and Scientists	CRC Press, Taylor & Francis Group		2018	Lesson 2
Chapra, S.C.	Applied Numerical Methods with MATLAB for Engineers and Scientists	Fourth edition, McGraw-Hill		2017	Lesson 2
Chapra, S.C., and Canale, R.P.	Numerical Methods for Engineers	Eight edition, McGraw-Hill	978-1-260-23207-3	2021	Lessons 3, 4, 5 and 6
Chaskalovic, J.	Mathematical and Numerical Methods for Partial Differential Equations: Applications for Engineering Sciences	Springer		2014	Lesson 5
Cheney, W., and Kincaid, D.	Numerical Mathematics and Computing	Seventh edition, Cengage Learning		2013	Lessons 3, 4, 5 and 6
Kharab, A. and Guenther, R.B	An Introduction to Numerical Methods: A MATLAB Approach	CRC Press, Taylor & Francis Group	978-1-138-09307-2	2019	Lessons 2-6
Epperson, J.F.	An Introduction to Numerical Methods and Analysis	John Wiley & Sons		2013	Lessons 3 and 4
Whiteley, J.	Finite Element Methods: A Practical Guide	Springer		2017	Lesson 5
Kochenderfer, M.J., and Wheeler, T.A.	Algorithms for Optimization	Massachusetts Institute of Technology Press		2019	Lesson 6
Gander, W., Gander, M.J., and Kwok, F.	Scientific Computing: An Introduction using Maple and MATLAB	Fourth edition, Springer		2014	Lessons 3, 4 and 6
Gilat, A.	MATLAB: An Introduction with Applications	Fifth edition, John Wiley & Sons		2014	Lesson 2
Giordano, F.R., Fox, W.P., and Horton, S.B.	A First Course in Mathematical Modeling	Fifth edition, Brooks/Cole Cengage Learning		2014	Lesson 1
Surana, K.S.	Numerical Methods and Methods of Approximation in Science and Engineering	CRC Press, Taylor & Francis Group	978-0-367-13672-7	2019	Lessons 3 and 4
Eck, C., and Garcke, H., and Knabner, P.	Mathematical Modeling	Springer	978-3-319-55161-6	2017	Lesson 1
Holmes, M.H.	Introduction to Scientific Computing and Data Analysis	Springer		2016	Lessons 3, 4 and 6
Kiusalaas, J.	Numerical Methods in Engineering with MATLAB	Third edition, Cambridge		2016	Lessons 3, 4 and 6

Lindfield, G.R., and Penny, J.E.T.	Numerical Methods using MATLAB	University Press Fourth Edition, Academic Press, Elsevier	978-0-12-812256-3	2019	Lessons 3 and 6
Lyche, T., and Merrien, JL.	Exercises in Computational Mathematics with MATLAB	Springer-Verlag		2014	Lesson 2
Miller, G.	Numerical Analysis for Engineers and Scientists	Cambridge University Press		2014	Lesson 3
Esfandiari, R.S.	Numerical Methods for Engineers and Scientists Using MATLAB	Second edition, CRC Press, Taylor & Francis Group	978-1-4987-7742-1	2017	Lessons 2-5
Pedregal, P.	Optimization and Approximation	Springer	978-3-319-64842-2	2017	Lesson 6
Quarteroni, A., Saleri, A., and Gervasio, P.	Scientific Computing with MATLAB and Octave	Fourth edition, Springer-Verlag		2014	Lessons 3 and 6
Rao, S.S.	Engineering Optimization: Theory and Practice	Fifth edition, John Wiley & Sons	978-1119454816	2019	Lesson 6
Rao, S.S.	The Finite Element Method in Engineering	Sixth edition, McGraw-Hill	978-0-12-811768-2	2018	Lesson 5